NORMAL DISTRIBUTION

In the previous part of this chapter, you have learnt about Binomial distribution and Poisson distribution. These distributions are of discrete variables. Now you will learn the probability distribution of continuous variable.

A random variable is called **non-discrete** or **continuous** random variable if it can assume infinite number of values in a interval. For example, height and weight of people, scientific measurements, amount of rainfall etc. It is widely used in statistics. The probability distribution of a continuous random variable is called continuous probability distribution.

Probability Density Function of a Continuous Random Variable

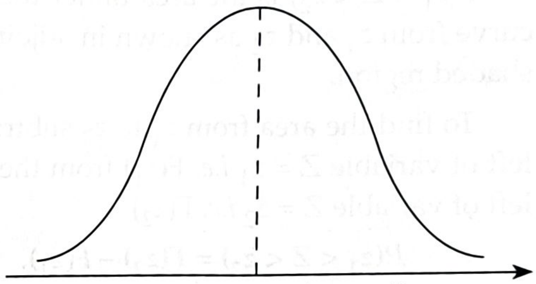
Let X be a continuous random variable. A function $y = f(x)$ is called **probability density function** of X if it satisfies the following properties:

(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ (iii) $P(a \leq X \leq b) = \int_a^b f(x)\,dx$.

Normal distribution

The most important continuous probability distribution in statistics is **normal probability distribution** or **normal distribution**. It is also called **Gaussian distribution.** In class XI, you have come across the normal distribution curve. It is a bell shaped curve symmetric about a vertical line through the mean. For normal distribution curve,

mean = mode = median



mean = mode = median

Normal curve

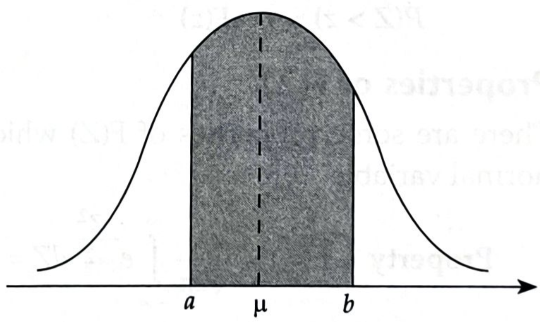
Normal curve never crosses the x-axis and extends indefinitely on right hand and left hand side. The total area under the normal curve is 1.

The density function of a normal distribution is called **normal density function** and is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$$

where parameters μ and σ are mean and standard deviation of X respectively.

The area under the normal curve bounded by the two ordinates $x = a$ and $x = b$ is equal to the probability of random variable X between $x = a$ and $x = b$. The probability $P(a < X < b)$ of a random variable X for normal distribution is shown in the adjoining figure by the shaded region.



Standard Normal Distribution

A random variable that has a normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$ is said to have **standard normal probability distribution.** The random variable in case of standard normal distribution is represented by Z.

So, the density function of the standard normal variable Z is given by

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}}, -\infty < Z < \infty$$

It is called **standard normal density function.**

Area under Standard Normal Curve

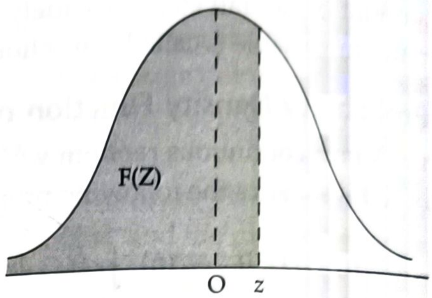
Like normal curves, the area under the standard normal curve also represents the probability. We need to calculate three types of probabilities:

(i) $P(Z < z)$ (ii) $P(z_1 < Z < z_2)$ (iii) $P(Z > z)$.

To calculate the probabilities, we define a **cumulative distribution function** $F(Z)$.

For any $Z \in (-\infty, \infty)$, we define

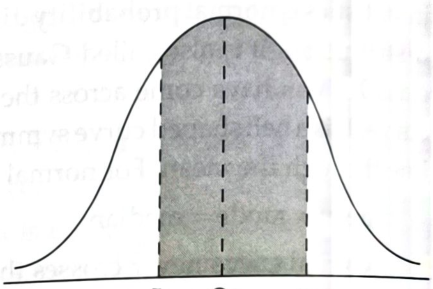
$$F(Z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}} dZ.$$



The function $F(Z)$ represents the entire area under standard normal curve which is on the left of the line $Z = z$.

So, $P(Z < z) = F(z)$

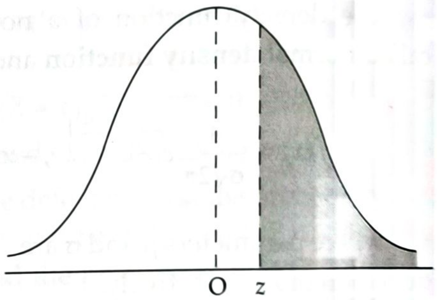
The values of $F(Z)$ have been computed and are given in the form of a table at the end of this book. This table can be used to find the probabilities of a standard normal variable.



$P(z_1 < Z < z_2)$ is the area under the standard normal curve from z_1 and z_2 as shown in adjoining figure by the shaded region.

To find the area from z_1 to z_2 subtract the area to the left of variable $Z = z_1$ *i.e.* $F(z_1)$ from the entire area to the left of variable $Z = z_2$ *i.e.* $F(z_2)$

$\therefore$ $P(z_1 < Z < z_2) = F(z_2) - F(z_1)$.



$P(Z > z)$ is the area under the standard normal curve which is to the right of the line $Z = z$ as shown in adjoining figure by the shaded region.

As the entire area under the normal curve is 1, so to find the area to the right of the line $Z = z$, subtract the area to the left of the line $Z = z$ *i.e.* $F(z)$ from 1.

$\therefore$ $P(Z > z) = 1 - P(Z \le z)$

$P(Z > z) = 1 - F(z)$

Properties of F(Z)

There are some properties of $F(Z)$ which are useful to compute the probabilities of a standard normal variable.

Property 1. $F(\infty) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{Z^2}{2}} dZ = 1$

Clearly, $F(\infty)$ represents the total area under the standard normal curve which is 1.

So, $F(\infty) = 1$.

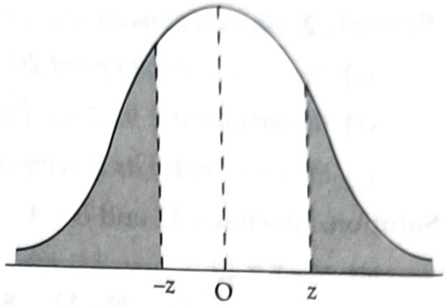
Property 2. $F(0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\frac{Z^2}{2}} dZ = 0.5$

Since standard normal curve is symmetric about mean $\mu = 0$, so $F(0)$ represents the area on the left of 0 which is half the area under the curve.

So, $F(0) = \frac{1}{2} = 0.5$

Property 3. $F(-z) = 1 - F(z)$

$F(-z)$ represents the area to the left of the line $Z = -z$. By the symmetry, this area is equal to the area to the right of the line $Z = z$ as shown in adjoining figure.



So, $F(-z) = P(Z > z)$

$F(-z) = 1 - P(Z \le z)$

$\therefore$ $F(-z) = 1 - F(z)$.

Area under any Normal Curve

You have learnt to find the area under standard normal curve. To find the area under any normal curve first convert the normal distribution random variable X to the standard normal distribution variable Z by using the formula

$$Z = \frac{X - \mu}{\sigma}$$

where μ is mean and σ is standard deviation. After converting the normal variable X to the standard normal variable Z, find the area using cumulative distribution function *i.e.* F(z) from table.

Remark

If X is a continuous random variable, then the probability that X lies in some interval is not affected by whether the end points of the interval are included or not *i.e.*

$P(X < a) = P(X \le a)$; $P(X > b) = P(X \ge b)$

$P(a < X < b) = P(a \le X \le b) = P(a < X \le b) = P(a \le X < b)$

ILLUSTRATIVE EXAMPLES

Example 1. *Calculate Z-score for a normal distribution of length of 7 rare species of Indian butterfly that you have in your garden.*

Butterfly	1	2	3	4	5	6	7
Length (in cm)	2	2	3	2	5	1	6

Solution. Mean $\mu = \frac{2+2+3+2+5+1+6}{7} = \frac{21}{7} = 3$

and $\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2}{n}}$

$\Rightarrow \quad \sigma = \sqrt{\frac{(2-3)^2 + (2-3)^2 + (3-3)^2 + (2-3)^2 + (5-3)^2 + (1-3)^2 + (6-3)^2}{7}}$

$= \sqrt{\frac{1+1+0+1+4+4+9}{7}} = \sqrt{\frac{20}{7}}$

$= \sqrt{2.857} = 1.69$

Butterfly	1	2	3	4	5	6	7
Length (in cm)	2	2	3	2	5	1	6
$Z = \frac{x-\mu}{\sigma}$	$\frac{2-3}{1.69}$ $= -0.59$	$\frac{2-3}{1.69}$ $= -0.59$	$\frac{3-3}{1.69} = 0$	$\frac{2-3}{1.69}$ $= -0.59$	$\frac{5-3}{1.69}$ $= 1.18$	$\frac{1-3}{1.69}$ $= -1.18$	$\frac{6-3}{1.69}$ $= 1.78$

Example 2. *Given that mean of a normal variable X is 12 and standard deviation is 4, then find*

(*a*) *the Z-score of data point 20.*

(*b*) *the data point if its Z-score is 5.*

(*c*) *the data point if its Z-score is –2.*

Solution. Given $\mu = 12$ and $\sigma = 4$

(*a*) $\because \ x = 20$

$$\therefore \ Z = \frac{x - \mu}{\sigma} = \frac{20 - 12}{4} = \frac{8}{4} = 2$$

(*b*) $\because \ Z = 5$

$$\therefore \ 5 = \frac{x - 12}{4} \Rightarrow x - 12 = 20$$

$$\Rightarrow x = 32$$

(*c*) $\because \ Z = -2$

$$\therefore \ -2 = \frac{x - 12}{4} \Rightarrow x - 12 = -8$$

$$\Rightarrow x = 4.$$

Example 3. *If Z is a standard normal variable, find each of the following probabilities:*

(*i*) $P(Z < 1.5)$ (*ii*) $P(Z < 1.23)$ (*iii*) $P(Z > 0.83)$ (*iv*) $P(Z < -1.05)$

(*v*) $P(2.15 < Z < 3.07)$ (*vi*) $P(Z > -1.83)$ (*vii*) $P(-2.38 < Z < -1.45)$

(*viii*) $P(-0.78 < Z < 1.87)$

Solution. (*i*) $P(Z < 1.5) = F(1.5) = 0.9332$

(see row against 1.5 under column .00 of the cumulative probability table)

(*ii*) $P(Z < 1.23) = F(1.23)$

$= 0.8907$

(see row against 1.2 under column .03 of the cumulative probability table)

$(\because P(Z > z) = 1 - P(Z \le z))$

(*iii*) $P(Z > 0.83) = 1 - P(Z \le 0.83)$

$= 1 - F(0.83)$

$= 1 - 0.7967 = 0.2033$ (see row against 0.8 under column .03 of the table)

(*iv*) $P(Z < -1.05) = F(-1.05)$

$= 1 - F(1.05)$ (using property 3)

$= 1 - 0.8531 = 0.1469$ (see row against 1.0 under column .05 of the table)

$(\because P(z_1 < Z < z_2) = F(z_2) - F(z_1))$

(*v*) $P(2.15 < Z < 3.07) = F(3.07) - F(2.15)$

$= 0.9989 - 0.9842 = 0.0147$

(*vi*) $P(Z > -1.83) = 1 - P(Z \le -1.83) = 1 - F(-1.83) = 1 - [1 - F(1.83)]$ (using property 3)

$= F(1.83) = 0.9664$

(*vii*) $P(-2.38 < Z < -1.45) = F(-1.45) - F(-2.38)$

$= [1 - F(1.45)] - [1 - F(2.38)]$

$= F(2.38) - F(1.45)$

$= 0.9913 - 0.9265 = 0.0648$

(*viii*) $P(-0.78 < Z < 1.87) = F(1.87) - F(-0.78)$

$= F(1.87) - [1 - F(0.78)]$

$= 0.9693 - [1 - 0.7823]$

$= 0.9693 - 0.2177 = 0.7516$

Example 4. *If X is a normal distribution random variable with mean $\mu = 10$ and standard deviation $\sigma = 2$, find each of the following probabilities:*

(i) $P(X < 13)$ (ii) $P(X > 16)$ (iii) $P(12 < X < 14.5)$

(iv) $P(X > 8)$ (v) $P(X < 9.8)$

Solution. Given $\mu = 10, \sigma = 2$

So, $Z = \dfrac{X - 10}{2}$

(i) For $X = 13$, $Z = \dfrac{13 - 10}{2} = 1.5$

$\therefore$ $P(X < 13) = P(Z < 1.5)$

$= F(1.5) = 0.9332$

(ii) For $X = 16$, $Z = \dfrac{16 - 10}{2} = 3$

$\therefore$ $P(X > 16) = P(Z > 3) = 1 - P(Z \leq 3) = 1 - F(3)$

$= 1 - 0.9986 = 0.0014$

(iii) For $X = 12$, $Z = \dfrac{12 - 10}{2} = 1$,

for $X = 14.5$, $Z = \dfrac{14.5 - 10}{2} = 2.25$

$\therefore$ $P(12 < X < 14.5) = P(1 < Z < 2.25) = F(2.25) - F(1)$

$= 0.9878 - 0.8413 = 0.1465$

(iv) For $X = 8$, $Z = \dfrac{8 - 10}{2} = -1$

$\therefore$ $P(X > 8) = P(Z > -1) = 1 - P(Z \leq -1)$

$= 1 - F(-1) = 1 - [1 - F(1)]$

$= F(1) = 0.8413$

(v) For $X = 9.8$, $Z = \dfrac{9.8 - 10}{2} = -0.1$

$\therefore$ $P(X < 9.8) = P(Z < -0.1) = 1 - F(0.1)$

$= 1 - 0.5398 = 0.4602$

Example 5. *The marks obtained in a certain examination follow normal distribution with mean 30 and standard deviation 10. If 1000 students appeared in the examinations, calculate the number of students scoring*

(i) *less than 33 marks*

(ii) *more than 50 marks*

(iii) *between 30 and 45 marks.*

Solution. Let X denote the marks obtained in the examination.

Given $\mu = 30, \sigma = 10$, then $Z = \dfrac{X - 30}{10}$.

(i) $P(X < 33) = P\left(Z < \dfrac{33 - 30}{10}\right) = P(Z < 0.3)$

$= F(0.3) = 0.6179$

$\therefore$ Number of students scoring less than 33 marks

$= 1000 \times 0.6179 = 617.9$ *i.e.* 618.

Hence, 618 students scored less than 33 marks.

(ii) $P(X > 50) = P\left(Z > \frac{50-30}{10}\right) = P(Z > 2)$

$= 1 - P(Z \le 2) = 1 - F(2) = 1 - 0.9772$

$= 0.0228$

$\therefore$ Number of students scoring more than 50 marks

$= 1000 \times 0.0228 = 22.8$ *i.e.* 23.

Hence, 23 students scored more than 50 marks.

(iii) $P(30 < X < 45) = P\left(\frac{30-30}{10} < Z < \frac{45-30}{10}\right)$

$= P(0 < Z < 1.5) = F(1.5) - F(0)$

$= 0.9332 - 0.5 = 0.4332$

$\therefore$ Number of students scoring between 30 and 45 marks

$= 1000 \times 0.4332 = 433.2$ *i.e.* 433.

Hence, 433 students scored between 30 and 45 marks.

Example 6. *In a district exam scores of 300 students of class XII are recorded at the end of the session.*

(a) *Ramesh scored 800 marks out of 1000. The average score for the batch was 700 and the standard deviation was calculated to be 180. Find out how has Ramesh scored compared to his batchmates in the whole district.*

(b) *Sudha scored 420 marks in the same batch. What can you say about her performance as compared to the batch of 300 students?*

(c) *How much has Abhay scored if he has done better than 44.83% of his batchmates?*

Solution. Let X denote the marks scored in the district exam.

Given $\mu = 700$, $\sigma = 180$, then $Z = \frac{X - 700}{180}$.

(a) $P(X < 800) = P\left(Z < \frac{800-700}{180}\right) = P(Z < 0.56)$

$= F(0.56) = 0.7123$

$\therefore$ Number of students who scored less than 800 marks

$= 300 \times 0.7123 = 213.69$ *i.e.* 214

Hence, we can say that Ramesh did better than 214 students.

(b) $P(X < 420) = P\left(Z < \frac{420-700}{180}\right) = P(Z < -1.56)$

$= F(-1.56) = 0.0594$

$\therefore$ Number of students who scored less than 420 marks

$= 300 \times 0.0594 = 17.82$ *i.e.* 18.

Hence, we can say that Sudha did better than 18 students.

(c) Given that Abhay has done better than 44.83 students, then Z-score corresponding to 44.83% *i.e.* 0.4483 in the table $= -0.13$.

So, $-0.13 = \frac{X - 700}{180} \Rightarrow X - 700 = -23.4$

$\Rightarrow X = 676.6$ *i.e.* 677.

Hence, Abhay has scored approximately 677 marks out of 1000.

Example 7. *Given that the scores of a set of candidates on an IQ test are normally distributed. If the I.Q. test has a mean of 100 and a standard deviation of 10, what is the probability that a candidate who takes the test will score between 90 and 110?*

Solution. Given $\mu = 100$ and $\sigma = 10$.

So, required probability $= P(90 < X < 110)$

$$= P\left(\frac{90-100}{10} < Z < \frac{110-100}{10}\right)$$

$$= P(-1 < Z < 1)$$

$$= F(1) - F(-1) = F(1) - [1 - F(1)]$$

$$= 2F(1) - 1 = 2 \times 0.8413 - 1 = 0.6826$$

Example 8. *In a normal distribution 31% of the articles are under 45 and 8% are over 64. Calculate the mean and standard deviation of the distribution.*

Solution. Let X be a normal distribution random variable, then

$P(X < 45) = 31\%$ *i.e.* 0.31

and $P(X > 64) = 8\%$ *i.e.* 0.08

Let the mean and the standard deviation of the distribution be μ and σ respectively, then

for $X = 45$, $Z = \dfrac{45-\mu}{\sigma}$ and

for $X = 64$, $Z = \dfrac{64-\mu}{\sigma}$

$$\therefore \quad P(X < 45) = P\left(Z < \frac{45-\mu}{\sigma}\right) = 0.31$$

$$\Rightarrow \quad P\left(Z < \frac{45-\mu}{\sigma}\right) = P(Z < -0.5) \qquad \text{(using table)}$$

$$\Rightarrow \quad \frac{45-\mu}{\sigma} = -0.5 \qquad \ldots(i)$$

$$\text{and } P(X > 64) = P\left(Z > \frac{64-\mu}{\sigma}\right) = 0.08 = 1 - 0.92$$

$$\Rightarrow \quad P\left(Z > \frac{64-\mu}{\sigma}\right) = 1 - P(Z \le 1.4) = P(Z > 1.4) \qquad \text{(using table)}$$

$$\Rightarrow \quad \frac{64-\mu}{\sigma} = 1.4 \qquad \ldots(ii)$$

Dividing equation (*i*) by (*ii*), we get

$$\frac{45-\mu}{64-\mu} = \frac{-0.5}{1.4} \Rightarrow 63 - 1.4\mu = -32 + 0.5\mu$$

$$\Rightarrow \quad 95 = 1.9\mu \Rightarrow \mu = 50.$$

Substituting $\mu = 50$ in equation (*i*), we get

$$\frac{45-50}{\sigma} = -0.5 \Rightarrow \sigma = 10.$$

Hence, mean 50 and standard deviation $= \sigma = 10$.

Example 9. *The weights of students of class XII follow normal distribution with mean 50 kg and standard deviation 2 kg. Find the probability that a student selected at random will have weight*

(i) less than 45 kg

(ii) more than 54 kg

(iii) between 48 and 56 kg.

Solution. Let X denote the weight of the students of class XII.

Given $\mu = 50, \sigma = 2$, then $Z = \dfrac{X-50}{2}$.

$$(i)\ P(X < 45) = P\left(Z < \frac{45-50}{2}\right) = P(Z < -2.5)$$

$$= F(-2.5) = 1 - F(2.5) = 1 - 0.9938$$

$$= 0.0062$$

$$(ii)\ P(X > 54) = P\left(Z > \frac{54-50}{2}\right) = P(Z > 2)$$

$$= 1 - P(Z \le 2) = 1 - F(2) = 1 - 0.9772$$

$$= 0.0228$$

$$(iii)\ P(48 < X < 56) = P\left(\frac{48-50}{2} < Z < \frac{56-50}{2}\right)$$

$$= P(-1 < Z < 3) = F(3) - F(-1)$$

$$= F(3) - [1 - F(1)] = 0.9986 - [1 - 0.8413]$$

$$= 0.9986 - 0.1587 = 0.8399$$

Example 10. *The monthly salaries of workers in a certain factory are normally distributed. The mean salary is ₹ 4000 and standard deviation is ₹ 450. If 668 workers are getting salary less than ₹ 3325, find the total number of workers in the factory.*

Solution. Let the total number of workers in the factory be n.

Given $\mu = 4000, \sigma = 450$, then $Z = \dfrac{X-4000}{450}$.

$$P(X < 3325) = P\left(Z < \frac{3325-4000}{450}\right)$$

$$= P(Z < -1.5) = F(-1.5) = 1 - F(1.5)$$

$$= 1 - 0.9332 = 0.0668$$

According to given, $0.0668 \times n = 668$

$$\Rightarrow \quad n = \frac{668}{0.0668} = \frac{668}{668} \times 10000 = 10000$$

Hence, total number of workers is 10000.

Example 11. *If X is normally distributed with mean 10 and standard deviation 4, find x such that the probability of X between 10 and x is 0.4772.*

Solution. Given $\mu = 10, \sigma = 4$.

Also, $P(10 < X < x) = 0.4772$,

then $Z = \dfrac{X - 10}{4}$.

$P(10 < X < x) = 0.4772$

$\Rightarrow \quad P\left(\dfrac{10-10}{4} < Z < \dfrac{x-10}{4}\right) = 0.4772$

$\Rightarrow \quad P\left(0 < Z < \dfrac{x-10}{4}\right) = 0.4772$

$\Rightarrow \quad F\left(\dfrac{x-10}{4}\right) - F(0) = 0.4772$

$\Rightarrow \quad F\left(\dfrac{x-10}{4}\right) - 0.5 = 0.4772$ (using property 2)

$\Rightarrow \quad F\left(\dfrac{x-10}{4}\right) = 0.9772$

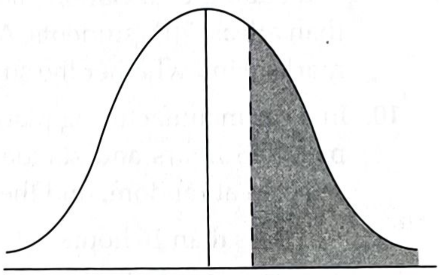
$\Rightarrow \quad \dfrac{x-10}{4} = 2$ (using table)

$\Rightarrow \quad x = 18.$

Example 12. *It is known from the past experience that the number of telephone calls made daily in a certain community between 3 p.m. and 4 p.m. has a mean of 352 and standard deviation of 31. What percentage of the time will there be more than 400 telephone calls made in the community between 3 p.m. to 4 p.m.? (Use: $P(0 \le Z \le 1.55) = 0.4394$)* *(CBSE 2022 Term I)*

Solution. Given $\mu = 352$, $\sigma = 31$ and $X = 400$, then

$$Z = \frac{X - \mu}{\sigma} = \frac{400 - 352}{31} \Rightarrow Z = 1.55$$



So, the required probability $= P(X > 400) = P(Z > 1.55)$

Since area under the normal curve right to mean 0 is 0.5

$\therefore$ The required probability $= P(Z > 1.55)$

$= 0.5 - P(0 \le Z \le 1.55)$

$= 0.5 - 0.4394 = 0.0606$

Hence, the required percentage of the time = 6.06%

EXERCISE 9.6

1. If Z is a standard normal variable, find each of the following probabilities:

   (*i*) $P(Z < 1.9)$ (*ii*) $P(Z > 2.7)$ (*iii*) $P(Z < -0.84)$

   (*iv*) $P(0 < Z < 1.45)$ (*v*) $P(Z > -0.57)$ (*vi*) $P(-1.3 < Z < 2.4)$

   (*vii*) $P(1.8 < Z < 2.9)$ (*viii*) $P(-3.01 < Z < -1.38)$

2. If X is a normal distribution random variable with mean 12 and standard deviation 3, find each of the following probabilities:

   (*i*) $P(X < 15)$ (*ii*) $P(X \ge 18)$ (*iii*) $P(X < 9)$

   (*iv*) $P(X > 6)$ (*v*) $P(12 < X < 21)$

3. Given Z is a standard normal random variable, find Z for each of the following:

   (*i*) The area to the left of Z is 0.9750

   (*ii*) The area between 0 and Z is 0.4750

   (*iii*) The area to the right of Z is 0.1314 (*iv*) The area to the right of Z is 0.3300

(*v*) The area between –Z and Z is 0.9030.

4. For a certain type of laptops the charging time of batteries is normally distributed with mean 50 hours and standard deviation 15 hours. Utkarsh has one of these laptops. Find the probability that the charging time of battery will be between 50 and 70 hours.

5. The lifetime of an item produced by a machine is normally distributed with mean 12 months and standard deviation 2 months. Find the probability that an item produced by this machine will last

(*i*) less than 7 months (*ii*) between 7 and 12 months

(*iii*) more than 14 months.

6. If the heights of 1000 students are normally distributed with mean 173 cm and standard deviation 7.5 cm, how many students have heights

(*i*) greater than 183 cm? (*ii*) less than 162.5 cm?

(*iii*) between 165 cm and 180 cm?

7. The monthly salaries of 500 employees of a company are normally distributed with mean ₹ 10000 and standard deviation ₹ 1500. How many employees have salaries

(*i*) less than ₹ 8000? (*ii*) more than ₹ 13000?

(*iii*) between ₹ 9000 and ₹ 14500?

8. A radar unit is installed to measure the speeds of cars on a highway. The speeds are normally distributed with mean 80 km/h and standard deviation 10 km/h. If a car is chosen at random, find the probability that car is running

(*i*) at less than 60 km/h (*ii*) at more than 100 km/h

(*iii*) between 90 km/h and 110 km/h.

9. The test scores of university entrance test are normally distributed with mean 500 marks and standard deviation 100 marks. A student can get admission if he scored marks better than atleast 70% students. A student is selected at random and known to have a score of 585 marks. Find whether the student will get admission in the university or not.

10. In a car manufacturing plant, the time taken to assemble a car is normally distributed with mean 18 hours and standard deviation 1.5 hours. If a car manufactured in that plant is selected at random, find the probability that it is assembled in

(*i*) less than 16 hours (*ii*) more than 19 hours

(*iii*) between 18 hours to 21 hours.

11. The monthly salaries of workers in a certain factory are normally distributed with mean ₹ 5000 and standard deviation ₹ 500. There are 14 workers getting more than ₹ 6500 per month. Find the total number of workers in the factory.

12. If X is normally distributed with mean 10 and standard deviation 1.5, find x such that $P(X > x) = 0.0038$.

13. A company conducted an I.Q. test for randomly selected 50 employees. Volunteer A scored 74 out of the possible 120 points. If the average I.Q. test score was recorded as 62 and the standard deviation was 11. How well did volunteer A perform in the test as compared to the other volunteers?

14. On an average, ceiling fan manufactured by Jagdeep Corporation lasts 300 days with a standard deviation of 50 days. Assuming that the ceiling fans life is normally distributed, what is the probability that a ceiling fan will last atmost 365 days?

15. In a math aptitude test, student scores are found to be normally distributed having mean as 45 and standard deviation 5. What percentage of students have scores

(*i*) more than the mean score?

(*ii*) between 30 and 50?

Answers

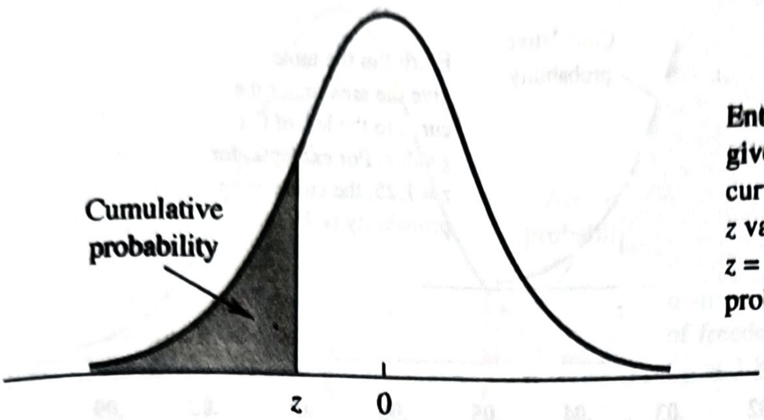
1. (i) 0.9713 (ii) 0.0035 (iii) 0.2005 (iv) 0.4265
(v) 0.7157 (vi) 0.895 (vii) 0.034 (viii) 0.0825
2. (i) 0.8413 (ii) 0.0228 (iii) 0.1587 (iv) 0.9772 (v) 0.4986
3. (i) 1.96 (ii) 1.96 (iii) 1.12 (iv) 0.44 (v) 1.66
4. 0.4082
5. (i) 0.0062 (ii) 0.4938 (iii) 0.1587
6. (i) 92 (ii) 81 (iii) 682
7. (i) 46 (ii) 11 (iii) 374
8. (i) 0.0228 (ii) 0.0228 (iii) 0.1573
9. Yes
10. (i) 0.0918 (ii) 0.2514 (iii) 0.4772
11. 10000
12. $x = 14$
13. A performed better than 43 employees
14. 0.9032
15. (i) 50% (ii) 84%

APPLICATIONS OF PROBABILITY DISTRIBUTIONS

The probability distributions are used to explain many real life events and describe them numerically. These are also used to predict future events. The distributions can be discrete or continuous depending upon the random variable. For example, the number of phone calls in call centres per minute is discrete variable whereas the amount of rainfall in a certain time period is a continuous variable. The following table shows the areas of applications of different probability distributions.

Distribution	Areas of applications
Binomial distribution	For quality control of a product, in many scientific works, military operations, memory test. In medical field such as effect of specific drug in a certain disease, the chance of survival of a patient after heart attack etc.
Poisson distribution	Number of typing errors in a page, number of phone calls in call centres per minute, number of cars passing through a certain point on a road, number of viruses that can effect a cell in a cell culture etc.
Normal distribution	In Social science, physical science, agriculture, medicine, engineering, errors correction. For interpretation of data obtained from experiments etc.

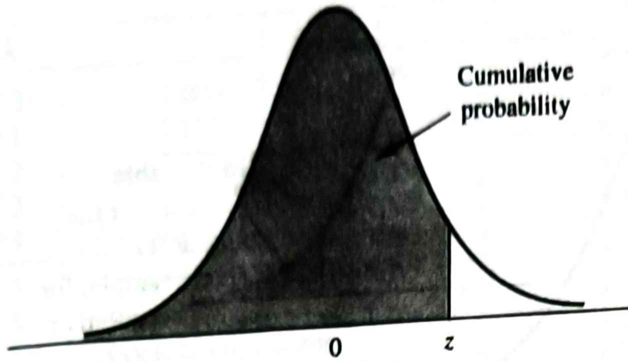
Cumulative Probabilities for the Standard Normal Distribution



Entries in the table give the area under the curve to the left of the z value. For example, for z = −.85, the cumulative probability is .1977.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Cumulative Probabilities for the Standard Normal Distribution (*Continued*)



Entries in the table give the area under the curve to the left of the z value. For example, for z = 1.25, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9913
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9986	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999